# Almond Hill Junior School


# CALCULATIONS POLICY

| | | |
|---|---|---|
| Governor committee: | Personal Development, Behaviour and Welfare Committee | |
| | Full governing body | |
| Policy due for review: | | |

**ALMOND HILL JUNIOR SCHOOL**

**CALCULATIONS POLICY FOR EMPLOYEES IN SCHOOLS**

This policy was reviewed January 2015

**The importance of mental mathematics**

While this policy focuses on written calculations in mathematics, we recognise the importance of the mental strategies and known facts that form the basis of all calculations. The following checklists outline the key skills and number facts that children are expected to develop throughout the school.

**To add and subtract successfully, children should be able to:**

- recall all addition pairs to 9 + 9 and number bonds to 10
- recognise addition and subtraction as inverse operations
- add mentally a series of one digit numbers (e.g. 5 + 8 + 4)
- add and subtract multiples of 10 or 100 using the related addition fact and their knowledge of place value (e.g. 600 + 700, 160 — 70)
- partition 2 and 3 digit numbers into multiples of 100, 10 and 1 in different ways (e.g. partition 74 into 70 + 4 or 60 + 14)
- use estimation by rounding to check answers are reasonable

**To multiply and divide successfully, children should be able to:**

- add and subtract accurately and efficiently
- recall multiplication facts to 12 x 12 = 144 and division facts to 144 ÷ 12 = 12
- use multiplication and division facts to estimate how many times one number divides into another etc.
- know the outcome of multiplying by 0 and by 1 and of dividing by 1
- understand the effect of multiplying and dividing whole numbers by 10, 100 and later 1000
- recognise factor pairs of numbers (e.g. that 15 = 3 x 5, or that 40 = 10 x 4) and increasingly able to recognise common factors
- derive other results from multiplication and division facts and multiplication and division by 10 or 100 (and later 1000)
- notice and recall with increasing fluency inverse facts
- partition numbers into 100s, 10s and 1s or multiple groupings
- understand how the principles of commutative, associative and distributive laws apply or do not apply to multiplication and division
- understand the effects of scaling by whole numbers and decimal numbers or fractions
- understand correspondence where n objects are related to m objects
- investigate and learn rules for divisibility

**For place value we use the following abbreviations:**

| | |
|---|---|
| Hundred thousands | HTH |
| Ten thousands | TTH |
| Thousands | TH |
| Hundreds | H |
| Tens | T |
| Ones | O |
| tenths | 1/10 |
| hundredths | 1/100 |
| thousandths | 1/1000 |

# Written methods for addition of whole numbers

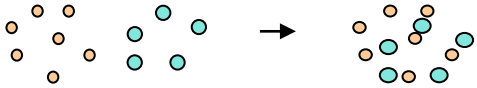
## Step 1 - Practical

**Combining two sets (aggregation)**

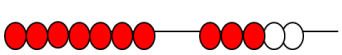
Putting together – two or more amounts or numbers are put together to make a total

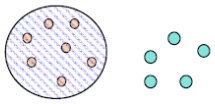
7 + 5 = 12

Count one set, then the other set. Combine the sets and count again. Starting at 1.



Counting along the bead bar, count out the 2 sets, then draw them together, count again. Starting at 1.

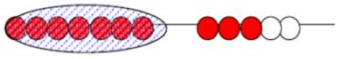




**Combining two sets (augmentation)**

*This stage is essential in starting children to calculate rather than counting*

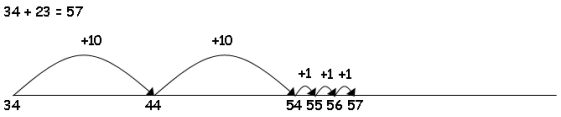
Where one quantity is increased by some amount. Count on from the total of the first set, e.g. put 3 in your head and count on 2. Always start with the largest number.



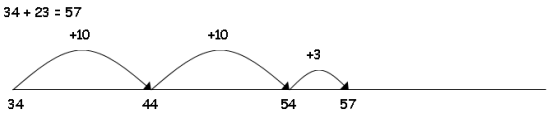

## Step 2 - The empty number line – linear addition

Children will begin to use 'empty number lines' themselves starting with the larger number and counting on.

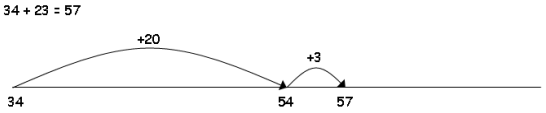
- ✓ First counting on in tens and ones



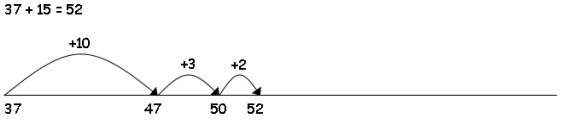

- ✓ Then helping children to become more efficient by adding the units in one jump (by using the known fact 4 + 3 = 7)



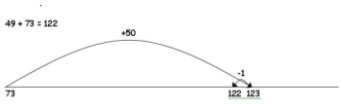

- ✓ Followed by adding the tens in one jump and the units in one jump.

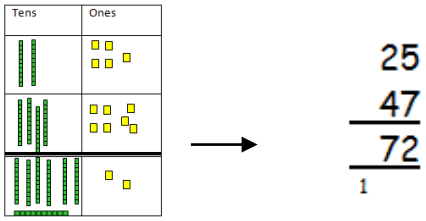



- ✓ Bridging through ten can help children become more efficient

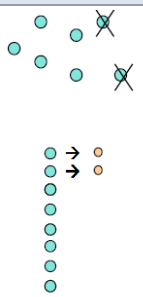
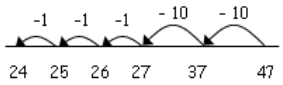
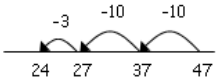
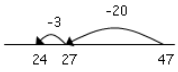
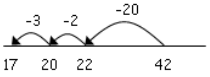


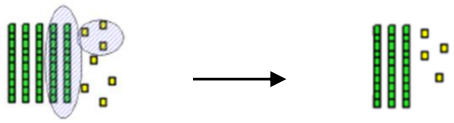

- ✓ Compensation



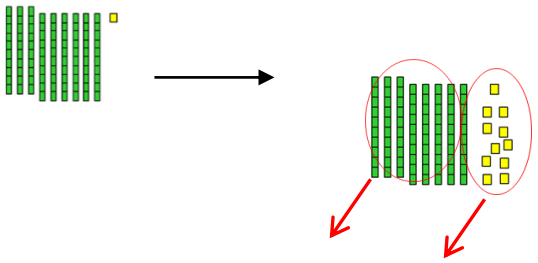
| Step 3 - Partitioning | |
|---|---|
| **Partitioning (Aggregation model)**<br>34 + 23 = 57<br>  <br>• Move on to a layout showing the addition of units to units and tens to tens separately. Start with adding units first to aid with standard column addition and avoid confusion.<br>• The addition of the tens in the calculation 47 + 76 is described in the words 'forty plus seventy equals one hundred and ten', stressing the link to the related fact 'four plus seven equals eleven'.<br>• The expanded method leads children to the more compact method so that they understand its structure and efficiency. | Write the numbers in columns.<br>Adding the units first:<br>**TO**<br>47<br>+ 76<br>13 ( 7 +6 )<br>110 ( 40 + 70 )<br>123 |
| **Step 4 – Formal compact method** | |
| **Gradation of difficulty- addition:**<br><br>1. No exchange<br>2. Extra digit in the answer<br>3. Exchanging ones to tens<br>4. Exchanging tens to hundreds<br>5. Exchanging ones to tens and tens to hundreds<br>6. More than two numbers in calculation<br>7. As 6 but with different number of digits<br>8. Decimals up to 2 decimal places (same number of decimal places)<br>9. Add two or more decimals with a range of decimal places | The use of diennes and formal written method to be introduced alongside each other to make links explicit.<br>  <br>**TO** 47 + 76 = 123 (1 1)<br>**HTO** 258 + 87 = 345 (1 1)<br>**HTO** 366 + 458 = 824 (1 1) |

## Written methods for subtraction of whole numbers

| **Step 1 – Practical** | |
|---|---|
| **Taking away (separation model)**<br>Where one quantity is taken away from another to calculate what is left. 7 – 2 = 5<br><br>**Finding the difference (comparison model)**<br>Two quantities are compared to find the difference.<br>8 – 2 = 6 |  |
| **Step 2 - : Using the empty number line** | |
| Children will begin to use empty number lines to support calculations.<br>**Counting back:**<br>✓ First counting back in tens and ones.<br><br>✓ Then helping children to become more efficient by subtracting the units in one jump (by using the known fact 4 + 3 = 7).<br><br>✓ Subtracting the tens in one jump and the units in one jump.<br><br>✓ Bridging through ten can help children become more efficient. | 47 – 23 = 24<br>  <br><br>47 – 23 = 24<br>  <br><br>47 – 23 = 24<br>  <br><br>42 – 25 = 17<br>  <br> |
| **Step 3 – Partitioning** | |

| | |
|---|---|
| **Take away (Separation model)**<br>57 – 23 = 34<br>Base 10 equipment:<br><br>Children remove the lower quantity from the larger set, starting with the ones and then the tens. In preparation for formal decomposition. |  |
| **Step 4 : Column method** | |

**Expanded decomposition:**
91 – 67 = 24



80 11

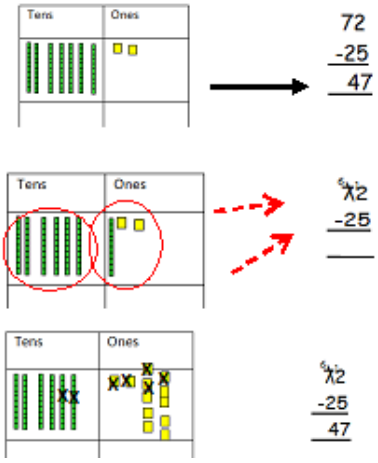
~~90~~ + ~~1~~
- 60 + 7
20 + 4

**Gradation of difficulty- subtraction:**

1. No exchange
2. Fewer digits in the answer
3. Exchanging tens for ones
4. Exchanging hundreds for tens
5. Exchanging hundreds to tens and tens to ones
6. As 5 but with different number of digits
7. Decimals up to 2 decimal places (same number of decimal places)
8. Subtract two or more decimals with a range of decimal places

**Compact decomposition**

The use of diennes and formal written method to be introduced alongside each other to make links explicit.



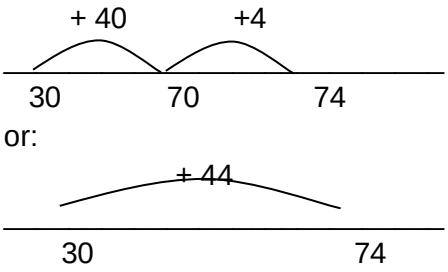

| T | U |
|---|---|
| 4 | 6 |
| -2 | 3 |
| 2 | 3 |

| T | U |
|---|---|
| $^{2}\cancel{3}^{1}$ | 1 |
| -2 | 7 |
| | 4 |

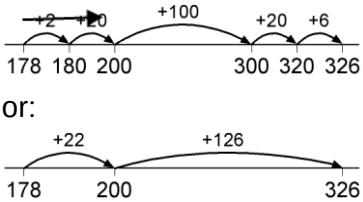
| H | T | U |
|---|---|---|
| $^{7}\cancel{8}$ | $^{1}7$ | 6 |
| - | 9 | 5 |
| 7 | 8 | 1 |

**The counting-up method**

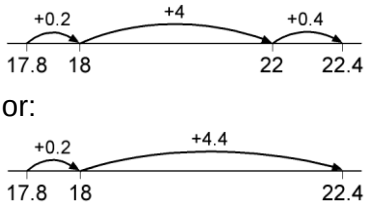
- The number of rows (or steps) can be reduced by combining steps. With two-digit numbers, this requires children to be able to work out the answer to a calculation such as 30 + ☐ = 74 mentally.



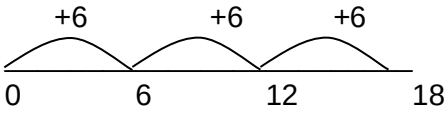
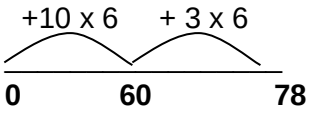
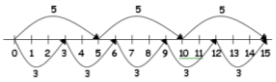
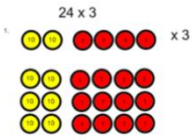

- With three-digit numbers the number of steps can again be reduced, provided that children are able to work out answers to calculations such as 178 + ☐ = 200 and 200 + ☐ = 326 mentally.

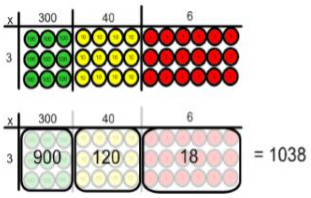



- The method can be used with decimals.

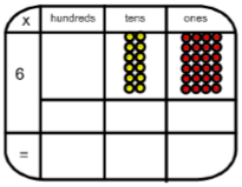


# Written methods for multiplication of whole numbers

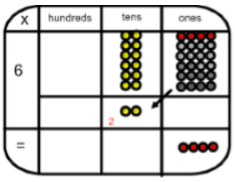
| **Step 1:** Repeated addition using a numberline | |
|---|---|
| • Use the empty numberline as in addition but add the same number over and over to represent the multiplication sum.<br>• This can be made more efficient by adding more than one multiple at a time | **3 x 6 = 18**<br>  <br>**13 x 6 = 78**<br>  <br>Children should know that 3 x 5 has the same answer as 5 x 3. This can also be shown on the number line.<br>  |
| **Step 2:** Grouping using arrays and scaling | |
| • Grouping in arrays using practical equipment to support.<br>• Ensuring pupils understand that the order does not affect the answer and making a link to times tables knowledge. (commutativity) |  <br>**3 x 6 = 6 x 3 = 18**<br>Children will also develop an understanding of **Scaling**<br>e.g. Find a ribbon that is 4 times as long as the blue ribbon<br>  |
| **Step 3 – Partitioning using grids** | |
| **Arrays leading into the grid method**<br>Children continue to use arrays and partitioning, where appropriate, to prepare them for the grid method of multiplication.<br>Arrays can be represented as 'grids' in a shorthand version and by using place value counters to show multiples of ten, hundred etc. |  |
| **Step 4 – Grid method** | |

| | |
|---|---|
| **<u>Grid method</u>**<br>This written strategy is introduced for the multiplication of TO x O to begin with. It may require column addition methods to calculate the total. |  <br><br><br> |
| **Step 5 – Short multiplication using place value counters** | |

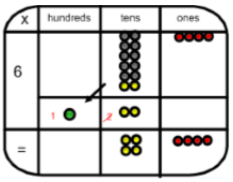
The use of place value counters and formal written method to be introduced alongside each other to make links explicit.



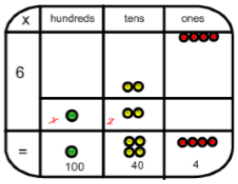

$$\begin{array}{r} 24 \\ \times\ 6 \\ \hline \end{array}$$




$$\begin{array}{r} 24 \\ \times\ 6 \\ \hline 4 \\ \hline \end{array}$$
2



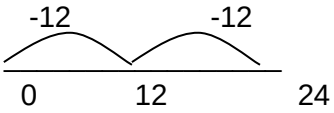

$$\begin{array}{r} 24 \\ \times\ 6 \\ \hline 44 \\ \hline \end{array}$$
1 2



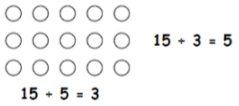

$$\begin{array}{r} 24 \\ \times\ 6 \\ \hline 144 \\ \hline \end{array}$$
1 2

| Step 6: Expanded short multiplication moving into short multiplication | |
|---|---|
| • The next step is to represent the method of recording in a column format and showing the working in brackets.<br>• Start with the units first as in other calculations – or from the right when using decimals.<br>• As soon as the children are confident with this they can move onto short multiplication.<br><br>• The recording is reduced further, with carry digits recorded below the line.<br>• If, after practice, children cannot use the compact method without making errors, they should return to the expanded format of stage 3.<br>• The step here involves adding 210 and 50 mentally with only the 5 in the 50 recorded.<br>**Gradation of difficulty (short multiplication)**<br>1. TO x O no exchange<br>2. TO x O extra digit in the answer<br>3. TO x O with exchange of ones into tens<br>4. HTO x O no exchange<br>5. HTO x O with exchange of ones into tens<br>6. HTO x O with exchange of tens into hundreds<br>7. HTO x O with exchange of ones into tens and tens into hundreds<br>8. As 4-7 but with greater number digits x O<br>9. O.t x O no exchange<br>10. O.t with exchange of tenths to ones<br>11. As 9 - 10 but with greater number of digits which may include a range of decimal places x O | **H T O**<br>38<br>X 7<br>56 ( 7 x 8)<br>210 ( 30 x 7)<br>266<br><br>**HTO**<br>**38**<br>**X 7**<br>**266**<br>5 |
| **Step 7: Long multiplication** | |
| • Reduce the recording further.<br>• Remove brackets as soon as not needed | 56 × 27 is approximately 60 × 30 = 1800.<br>**ThHTU**<br>5 6<br>x 2 7<br>3 $9_4$ 2 ( 56 x 7 )<br>1 $1_1$2 0 ( 56 x 20 )<br>1 5 1 2<br>1 |

## Written methods for division of whole numbers

| Step 1 - Sharing equally | |
|---|---|
| • Children start with a total number and use practical equipment to model sharing between the number being divided eg there are four children and 12 sweets. Share them out equally. One for you, one for you... The answer is the number in each group. | **sharing**  12÷4 = 3 |
| **Step 2: Repeated subtraction and grouping** | |
| • Children start with a total number and use practical equipment to model grouping the number being divided eg there are 12 sweets and they are put into groups of four. The answer is the number of groups.<br>• Use the empty numberline as in subtraction but subtract the same number over and over to represent the division sum.<br>• This can be made more efficient by subtracting more than one multiple at a time<br>• The answer is the number of multiples that have been subtracted<br>• First use without remainders and when competent with remainders<br>• Teach when to round up or down in a context. | 12÷4 = 3 **grouping**  **24 ÷ 12 = 2**   |
| **Step 3 – Arrays and partitioning** | |

Children learn to model a division calculation using an array. This model supports their understanding of the development of partitioning and the 'bus stop method' in a written method. This model also connects division to finding fractions of discrete quantities.



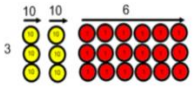

**Partitioning for division**

The array is also a flexible model for division of larger numbers

78 ÷ 3

78 ÷ 3 = (30 ÷ 3) + (30 ÷ 3) + (18 ÷ 3) =

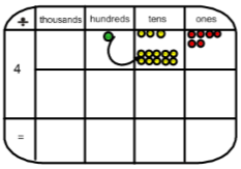
10 + 10 + 6 = 26

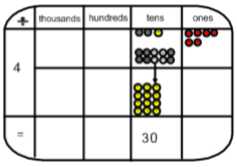


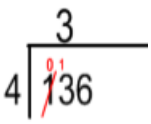

**Step 4 - Short division for TU ÷ U , TU ÷ TU and HTU ÷ TU**

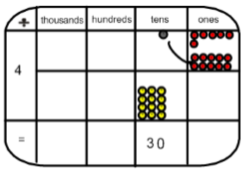
Begin to group place value counters into an array to show short division working

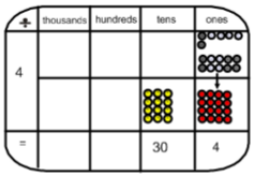
136 ÷ 4

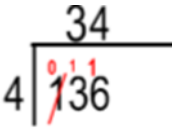


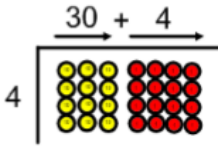



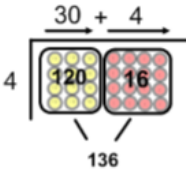



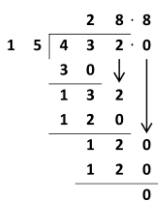




- First use without remainders and when competent with remainders
- Multiples of the divisor can be written down the side of the page to assist with the calculation. This will be especially useful when dividing by a TU number.

**T O**
**2 7**
**3 | 8²1**

1 - 3
2 - 6
3 -9

| **Step 5 – Long division** | |
|---|---|
| • First write multiples of the divisor across the top of the page to assist calculations.<br>• Look at first two digits of the dividend (in this case 43). How many lots of the divisor are there (2 x 15 = 30)? Write the quotient (2) above the 'bus stop' and the 30 (2 x 15) below the 43. Take the 30 from 43, leaving the remainder of 13<br>• Drop down the next digit of the dividend (in this case the 2) to sit next to the remainder from the previous step (the 13). We are now looking at 'how many 15s are there in 132'.<br>• Repeat the above process: 15 x 8 = 120. We write 120 below the 132 and subtract, leaving a remainder of 12.<br>• This could be left as a remainder.<br>• Alternatively, on the original dividend, add a decimal point and a 0. Drop the 0 down to sit next to the previous remainder of12, so it becomes 120.<br>• How many 15s are in 120? Answer: 8. This is written to the right of the decimal point in the quotient above the bus stop.<br>• Final answer: 28.8 | 15, 30, 45, 60, 75, 90, 105, 120, 135<br><br>432 ÷ 15 becomes<br><br>  <br><br><br>Answer: 28·8 |

***Remainders can be expressed as***

- Remainders
- Fractions
- Decimals
- Rounded up or down depending on real life context